BRAILLE MATHEMATICAL NOTATION

By H. M. Taylor

SECOND AMERICAN REVISION

1942

Compiled by

John B. Curtis, *Founder and Former Supervisor*
Classes for the Blind, Chicago Public Schools

Marjorie S. Hooper, *Braille Editor*
American Printing House for the Blind

Edward J. Waterhouse, *Instructor in Mathematics*
Perkins Institution and Massachusetts School
for the Blind

Published in both Braille and Ink Print

by the

AMERICAN PRINTING HOUSE FOR THE BLIND

Louisville, Kentucky

1 9 4 2


EX-
LIBRIS
AMERICAN FOUNDATION
FOR THE BLIND INC.

BRAILLE MATHEMATICAL NOTATION

By H. M. Taylor

SECOND AMERICAN REVISION

1942

Compiled by

John B. Curtis, *Founder and Former Supervisor*
Classes for the Blind, Chicago Public Schools

Marjorie S. Hooper, *Braille Editor*
American Printing House for the Blind

Edward J. Waterhouse, *Instructor in Mathematics*
Perkins Institution and Massachusetts School
for the Blind

Published in both Braille and Ink Print

by the

AMERICAN PRINTING HOUSE FOR THE BLIND

Louisville, Kentucky

1 9 4 2

Braille and ink-print copies of the
BRAILLE MATHEMATICAL CODE may be
obtained from the

AMERICAN PRINTING HOUSE FOR THE BLIND
Louisville, Kentucky

PREFACE

The 2nd American Revision of the BRAILLE MATHEMATICAL NOTATION was undertaken because of the inability of the previous revision to meet the demands of modern mathematics texts, and because it was necessary to bring the Mathematics Code in line with Standard English Braille. This revision is based on the code devised by H. M. Taylor, and revised in 1920 by the Sub-Committee on Mathematics of the Commission on Uniform Type for the Blind (John B. Curtis, Chairman; with Claudia Potter and Nellie G. McIntyre); accepted by the Commission on Uniform Type, June 23, 1920; and referred for consideration and approval to the International Sub-Committee on Uniform Type.

In making this revision, two purposes have been kept in mind: One, the need for a manual to aid in the teaching of the Braille Mathematics Code; and Two, the need for a comprehensive guide for stereotypists, transcribers and proofreaders of Braille mathematical works.

The order of the introduction of the mathematical symbols is designed to follow the order of their introduction in arithmetic, algebra, geometry, and higher mathematics. This order was retained from the previous revision with very little change.

The arrangement of the material has been changed from paragraph to outline form, and the entire format expanded to increase ease of identification of the symbols. The Braille edition is embossed in one-side print, and the pagings of the Braille edition have been indicated in the ink-print edition, for the convenience of sighted instructors.

A number of additional illustrations of usage of the symbols have been included, and the list of abbreviations has been increased. Many of the usages noted are not recommended for common use, but represent forms which the student may find in Braille mathematical textbooks embossed prior to this revision or in other countries. An attempt has been made to indicate the preferred usage.

The inclusion of a Table of Contents and the change to outline arrangement should greatly increase the usability of the NOTATION by embossers and proofreaders.

October, 1942

JOHN B. CURTIS
MARJORIE S. HOOPER
EDWARD J. WATERHOUSE

CONTENTS

MATHEMATICAL NOTATION

(Note: Bold-face numbers inclosed in brackets indicate the pagings of the Braille edition of **Mathematical Notation.**)

ARBITRARY SYMBOLS AND THEIR USES

NUMERALS

Numeral Sign ⠼ (dots 3-4-5-6). Numerals are expressed by placing the **Numeral Sign** immediately before the first ten signs of the alphabet:

Cardinal Numbers are written thus:

1 ⠼⠁ 23 ⠼⠃⠉ 405 ⠼⠙⠚⠑

In numbers of more than three figures, except when writing the date of the year, the **Comma** ⠂ (dot 2) is used to divide off the thousands. This rule is frequently disregarded in numbers less than 10,000, and the **Comma** should also be omitted from computation examples if its inclusion throws figures into the [2] wrong column. Examples:

4,500,000 ⠼⠙⠂⠑⠚⠚⠂⠚⠚⠚

1927 (year, and sometimes in figures) ⠼⠁⠊⠃⠛

Numbers joined by the **Hyphen** do not require a second **Numeral Sign** unless the number is divided at the end of a line after the **Hyphen** when the **Numeral Sign** should be repeated at the beginning of the following line. Examples:

pages 210-810 ⠼⠏⠁⠚⠤⠓⠁⠚

pages 210-
810 ⠼⠏⠁⠚⠤
⠓⠁⠚

When an **Integral Number** is divided at the end of a line, the **Hyphen** is placed either at the end of the first line or at the beginning of the second, or preferably both, and the **Numeral Sign** should not be repeated. Example:

210- ⠼⠃⠁⠚⠤
-810 ⠤⠓⠁⠚

The **Numeral Sign** is frequently omitted from mathematical tables without [3] explanation when its presence may easily be assumed. It should not, however, be omitted after a sign of operation in cases where the various parts of a problem are written on separate lines. For illustrations, see examples under the General Sign of Operation, page 12.

Double Numeral Sign ⠼⠼ (dots 3-4-5-6, 3-4-5-6) indicates that the **Numeral Sign** is omitted from all the numbers of the immediate group. For example:

Find the sum of the following: 135, 795, 846, 273, 657, and 394. Solution:

[4]

Considerable discretion should be employed in the use of the **Double Number Sign.** It should not be used unless more than two numbers are involved. It should also not be used in books below the level of the fourth grade

Note: The use of the continuous line of dots 2-5, 2-5, 2-5, etc., etc., to indicate the addition of the figures in the columns above is authorized by the **General Sign of Operation**, page 12.

Triple Numeral Sign ⠼⠼⠼ (dots 3-4-5-6, 3-4-5-6, 3-4-5-6) indicates that all of the Braille signs represented by the first ten letters of the alphabet which follow on the page, or until the **Close Triple Numeral Sign** ⠸⠸⠸ (dots 1-4-5-6, 1-4-5-6, 1-4-5-6) is met, represent Arabic numerals. **(Note:** These signs are seldom employed.) [5]

Ordinal Numbers are written by placing the proper ending immediately following the **Cardinal Number** without a space. Braille contractions should be employed for the suffix according to the grade of Braille being used. Examples:

1st (Braille grades 1 and 1½) ⠼⠁⠌ or 1st (grade 2) ⠼⠁⠌

2nd ⠼⠃⠌

3rd ⠼⠉⠌

4th (Braille grade 1) ⠼⠙⠌ or 4th (Braille grades 1½ and 2) ⠼⠙⠌

It is recommended that the forms "nd" and "rd" should be used instead of "d" to prevent confusion in reading the suffix as "the Cardinal Number 4."

Roman Numerals of one letter are preceded by the **Capital Sign** ⠠ (dot 6); those of more than one letter, by the **Double Capital Sign** ⠠⠠ [6] (dots 6, 6). For example:

I ⠠⠊ III ⠠⠠⠊⠊⠊

II ⠠⠠⠊⠊ IV ⠠⠠⠊⠙

LETTERS

Letter Sign ⠰ (dots 5-6). Single letters, when representing algebraic quantities, are preceded by the **Letter Sign.** Contrary to literary Braille usage, it is suggested that this rule also apply to the writing of the letter "a" in mathematical Braille. Examples:

a ⠰⠁ x ⠰⠭

b ⠰⠃ y ⠰⠽

When the letter is preceded or followed immediately by another character, the **Letter Sign** is omitted if it is perfectly clear that the letter cannot be interpreted as a number. Examples:

3x  xy 

3 ÷ a  (See page 11 for **Division Sign.**)

In algebra, whenever a combination of letters may be taken for a word, it is recommended that the **Letter Sign** precede such a combination. [7] Examples:

ab (which might be taken for about)

an

cd (for could)

When an algebraic quantity represented by one of the first ten letters of the alphabet has a numerical coefficient, the **Multiplication Sign** (see page 11) follows the number, that is, it follows the coefficient. For example:

7a 

2a + 2b = 18  (See page 10 for **Plus Sign.**)

See also rules for **Mixed Numbers,** page 18; **Literal Index Sign,** page 20; **Literal Suffix Sign,** page 22; and **Factorial Sign,** page 28.

Greek Letters are formed by placing dot 3  immediately before corresponding English letters.

For example:

Delta 

Epsilon 

Eta 

Theta  [8]

Omicron 

Pi 

Upsilon 

Phi 

Chi 

Psi 

Omega 

(For a complete list of **Greek Letters,** see page 37.)

Geometric Points are usually expressed by **Small Letters** in Braille, contrary to the ink-print practice of using **Capital Letters,** except where the same letter is used in both capital and small forms in the same exercise.

DECIMALS

Decimal Point . ⠲ (dots 4-6). When a decimal fraction is joined to a whole number, only one **Numeral Sign** (that before the whole number) is required. Examples:

8.5 ⠼⠓⠲⠢

.61 ⠲⠋⠁ or 0.61 ⠼⠚⠲⠋⠁ [9]

Recurring Decimal ⠨ (dots 4-5-6). Examples:

$4.\dot{7}$ ⠼⠙⠲⠶

$4.1\dot{6}$ ⠼⠙⠲⠁⠶

$.\dot{1}42857$ ⠲⠂⠲⠁⠃⠉⠙⠑⠶

SYMBOLS OF VALUE, MEASUREMENT, ETC.

When in ink print a symbol or a literal abbreviation of value or measurement follows a numeral, the corresponding literal abbreviation or its equivalent, without the **Abbreviation Point,** is placed in Braille before the **Numeral Sign.**

American Money:

Dollar Sign $ ⠈⠎ (dots 2-5-6, 3-4-5-6). In decimal coinage the **Decimal Point** is used in place of the second **Numeral Sign.** [10] Examples:

$10.03 ⠈⠎⠼⠁⠚⠲⠚⠉

$ .61 ⠈⠎⠲⠋⠁ or $0.61 ⠈⠎⠼⠚⠲⠋⠁

Note: In books embossed prior to the adoption of Standard English Braille in 1932, the sign ⠈⠎ (dots 1-4-5, 3-4-5-6) is used to express the **Dollar Sign.**

Cent Sign ¢ ⠉ (dots 1-4, 3-4-5-6) should only be used to express amounts less than a dime, including the decimal parts of a cent, or when indicated in the ink-print text. The sign for **Mills** ⠍ (dots 1-3-4, 3-4-5-6) is seldom used. Examples:

9¢ ⠼⠊⠉

.015¢ ⠼⠲⠚⠑⠉ rather than $ .00015 ⠈⠎⠲⠚⠚⠚⠑ [11]

Foreign Money:

Pound Sterling £ ⠇ (dots 1-2-3, 3-4-5-6).

Shilling / or s ⠎ (dots 2-3-4, 3-4-5-6).

Pence d ⠙ (dots 1-4-5, 3-4-5-6).

Guinea gs ⠛⠎ (dots 1-2-4-5, 2-3-4, 3-4-5-6).

Franc fr ⠋⠗ (dots 1-2-4, 1-2-3-5, 3-4-5-6)

Mark (Reich) rm ⠍⠗ (dots 1-2-3-5, 1-3-4, 3-4-5-6).

Peso P ⠏ (dots 1-2-3-4, 3-4-5-6). [12]

Rupee rs ⠗⠎ (dots 1-2-3-5, 2-3-4, 3-4-5-6).

Yen Y ⠽ (dots 1-3-4-5-6, 3-4-5-6).

Miscellaneous Symbols:

Acres a ⠁ (dot 1).

Barrels bbl ⠃ ⠃ ⠇ (dots 1-2, 1-2, 1-2-3).

Bushels bu ⠃ ⠉ (dots 1-2, 1-3-6).

Centimeters cm ⠒ ⠔ (dots 1-4, 1-3-4).

Cubic Inches cu. in. ⠒ ⠉ ⠢ ⠊ ⠖ (dots 1-4, 1-3-6, 3-5-6, 2-4, 1-3-4-5). This same form can be used in writing "cubic feet," "cubic yards," etc.

Days da ⠒ ⠁ (dots 1-4-5, 1).

Decimeters dm ⠒ ⠔ (dots 1-4-5, 1-3-4).

Degrees ° ⠒ ⠤ (dots 1-4-5, 1-2-4-5). The Degree Sign may [13] be used to indicate measurements of arcs and angles as well as of temperatures. Examples:

10° ⠂ ⠒ ⠴ ⠁ ⠚

x° ⠂ ⠒ ⠴ ⠔ usually written x degrees ⠴ ⠒ ⠴ ⠂ ⠒ ⠴ ⠒ ⠒ ⠴ ⠒ ⠒ ⠴ ⠒ ⠒ ⠴

Dozen dz ⠒ ⠲ (dots 1-4-5, 1-3-5-6).

Feet ft ′ ⠒ ⠶ (dots 1-2-4, 2-3-4-5).

Gallons gal ⠒ ⠁ ⠃ (dots 1-2-4-5, 1, 1-2-3).

Gills gi ⠒ ⠊ (dots 1-2-4-5, 2-4).

Grams gm ⠒ ⠔ (dots 1-2-4-5, 1-3-4).

Gross gro ⠛⠗⠕ (dots 1-2-4-5, 1-2-3-5, 1-3-5).

Hours hr ⠓⠗ (dots 1-2-5, 1-2-3-5).

Hundred Weight cwt ⠉⠺⠞ (dots 1-4, 2-4-5-6, 2-3-4-5).

Inches in ″ ⠔⠉ (dots 2-4, 1-3-4-5). The contracted form should never be used in mathematical Braille. [14].

Kilograms kg ⠅⠛ (dots 1-3, 1-2-4-5).

Kilometers km ⠅⠍ (dots 1-3, 1-3-4).

Meters m ⠍ (dots 1-3-4).

Miles mi ⠍⠊ (dots 1-3-4, 2-4).

Millimeters mm ⠍⠍ (dots 1-3-4, 1-3-4).

Minutes min ′ ⠍⠔⠉ (dots 1-3-4, 2-4, 1-3-4-5) or ⠍⠉ (dots 1-3-4, 1-3-4-5). The former is the preferred usage, and should never be written in contracted form in mathematical Braille. This sign may be used to indicate minutes of angles and arcs as well as of time.

Months mo ⠍⠕ (dots 1-3-4, 1-3-5).

Ounces oz ⠕⠵ (dots 1-3-5, 1-3-5-6).

Pecks pk ⠏⠅ (dots 1-2-3-4, 1-3).

Per Cent Sign % ⠤⠴ (dots 2-5, 1-2-3-4). Examples: [15]

5% ⠼⠑⠤⠴

x% ⠼⠭⠤⠴ usually written x per cent ⠼⠭ ⠏⠻ ⠉⠝⠞

Pints pt ⠏⠞ (dots 1-2-3-4, 2-3-4-5).

Pound Weight # or lb ⠇⠃ (dots 1-2-3, 1-2).

Quarters qr ⠟⠗ (dots 1-2-3-4-5, 1-2-3-5).

Quarts qt ⠟⠞ (dots 1-2-3-4-5, 2-3-4-5).

Rods rd ⠗⠙ (dots 1-2-3-5, 1-4-5).

Seconds ″ ⠠⠆ (dots 2-3-4, 1-4). The sign for **Seconds** may be used to indicate seconds of angles and arcs as well as of time.

Square Feet sq. ft. ⠠⠎⠟ ⠋⠞ (dots 2-3-4, 1-2-3-4-5, 2-5-6, 1-2-4, 2-3-4-5). This same form can be used in writing "square inches," "square yards," etc.

Tablespoons tbsp ⠠⠞⠃⠏ (dots 2-3-4-5, 1-2, 2-3-4, 1-2-3-4). [16]

Teaspoons tsp ⠠⠞⠎⠏ (dots 2-3-4-5, 2-3-4, 1-2-3-4).

Tons t ⠠⠞ (dots 2-3-4-5).

Weeks wk ⠠⠺⠅ (dots 2-4-5-6, 1-3).

Yards yd ⠠⠽⠙ (dots 1-3-4-5-6, 1-4-5).

Years yr ⠠⠽⠗ (dots 1-3-4-5-6, 1-2-3-5).

Additional **Symbols of Measurements** may be developed by placing the accepted abbreviation authorized by any standard dictionary before the numeral sign without a space, as in the examples given above. In this connection, it should be noted that the form ⠉ (dots 1-4) cannot be used for "cups" or "cans," since in Braille this character represents "cents." It is also suggested that no abbreviation be used for "section," since this is not often needed and might be confused with the abbreviation for [17] "seconds."

Usages:

In a sequence of related terms, only the **Abbreviation** for the larger quantity is used, and the lesser quantity, with the **Numeral Sign**, follows close up. Examples:

35°0′45″ 

4 yds, 2 ft, 3 in 

3 hrs, 20 min, 4 sec 

Exceptions to this rule occur in writing Decimal Coinage (see page 5) and in expressing a definite point of Time when the **Colon** (dots 2-5) should be placed between the hours and minutes and the **Numeral Sign** should not be repeated. Example:

3:20  [18]

ELEMENTARY SIGNS OF OPERATION

(For additional signs used in advanced mathematics, see page 25.)

It is recommended that **Signs of Operation** not be used in literary Braille books of below the junior high school level.

Plus Sign +  (dots 2-6). Examples:

7 + 3 

x + y 

Minus Sign —  (dots 3-5). Examples:

8 — 2 

x — y 

Plus-or-Minus Sign ± ⠖⠒ ⠒⠖ (dots 2-6, 3-5). Examples:

20 ± 10 ⠼⠚ ⠖⠒ ⠼⠁⠚ x ± y ⠭ ⠖⠒ ⠽ [19]

Minus-or-Plus Sign ∓ ⠒⠖ ⠖⠒ (dots 3-5, 2-6). Examples:

50 ∓ 30 ⠼⠑⠚ ⠒⠖ ⠼⠉⠚ x ∓ y ⠭ ⠒⠖ ⠽

Multiplication Sign × ⠦⠒ ⠒⠴ (dots 1-6). Examples:

60 × 5 ⠼⠋⠚ ⠦⠒ ⠼⠑ 8x × 7y ⠼⠓⠭ ⠦⠒ ⠼⠛⠽

Division Sign ÷ ⠒⠌⠒ ⠒⠌⠒ (dots 3-4, 3-4). Examples:

8765 ÷ 325 ⠼⠓⠛⠋⠑ ⠒⠌⠒ ⠼⠉⠃⠑

95xy ÷ 5x ⠼⠊⠑⠭⠽ ⠒⠌⠒ ⠼⠑⠭

The symbols)...(⠦⠒ ⠒⠴ (dots 1-3-5 and 2-4-6) may be used when it is desired to place the divisor before the dividend. For example:

25)8625(345 ⠼⠃⠑⠦⠼⠓⠋⠃⠑⠴⠼⠉⠙ or

) 345 ⠦⠼⠉⠙

25)8625 ⠼⠃⠑⠦⠼⠓⠋⠃⠑⠴ [20]

It is recommended that this form not be used in algebraic division, as the letter "o" and the "ow" sign are frequently met in such problems.

As an aid in illustrating the meaning of such terms as "numerator," "denominator," "over," etc., fractions and problems in division may be set up to follow the ink-print form. This practice may also be followed in embossing complicated division problems in algebra. It is recommended, however, that the

student be encouraged to master the use of the authorized **Division Sign** as soon as possible, so that the problems may be worked out on a continuous Braille line on the slate or Braille-writer. Examples:

$$\frac{1}{x+3} + \frac{1}{x+5}$$

[21]

Note: For use of the continuous line of dots 2-5, 2-5, 2-5, etc., etc., to indicate the division line in the examples above, see the **General Sign of Operation,** below. (See also the **Double General Sign of Operation,** page 14.)

Equality Sign = ⠶ (dots 2-5, 2-5). Examples:

$7 + 5 = 12$

$x + y = 10$

General Sign of Operation _____ ⠒⠒⠒⠒⠒ (dots 2-5, 2-5, 2-5, etc.) When the parts of a problem of Addition, Subtraction, Multiplication or Division are placed one immediately beneath the other, the **General Sign of Operation** (or line) is used to separate the various operations in working the problem. Care should be taken to keep **Letter Signs, Numeral Signs,** [22] **Symbols of Value and Measurement,** and **Signs of Operation** in their respective columns, so that these symbols will not be confused with the figures of a problem when reading up and down the columns. Examples:

Addition: 456

\+ 228

———

Ans. 684

Subtraction: 456

— 228

———

Ans. 228

Multiplication: 456

× 228

———

3648

912

912

———

103,968 [23]

Division: 228)456(2

456

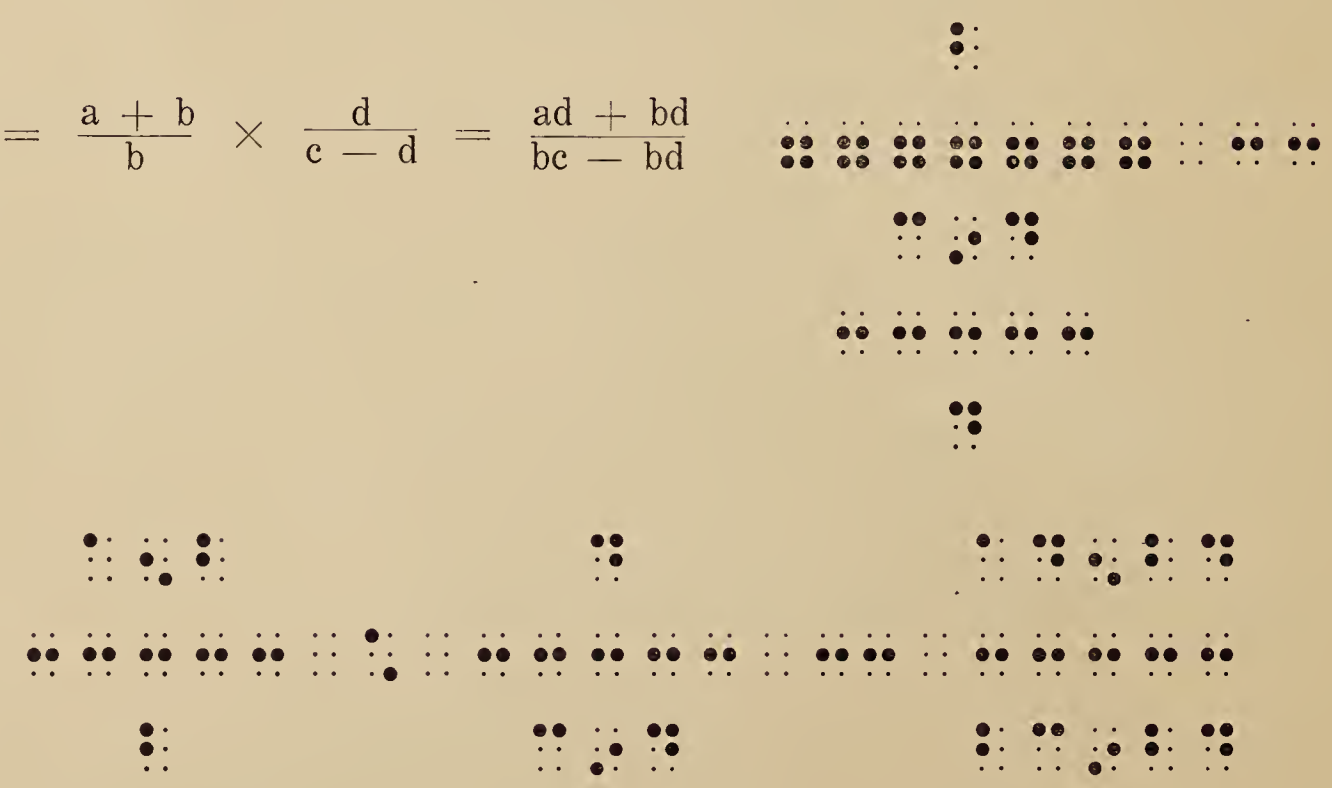
———

Care should be taken that the **General Sign of Operation** occupy not less than four Braille cells, lest it be confused with the **Equality Sign.**

Note: Students wishing to save time in working problems may substitute a skipped line in place of the **General Sign of Operation,** similar to the practice in working with an arithmetic slate. In such instances, the **Numeral Signs** and other **Signs of Operation** may also be omitted if their presence is readily understood.

Double General Sign of Operation ⠶⠶⠶⠶⠶⠶⠶ (dots 2-3-5-6, 2-3-5-6, 2-3-5-6, etc.) When it is desired to employ the ink-print set-up in embossing complicated problems in algebra composed of fractions or mixed numbers in either the numerator or [24] denominator, it is suggested that the **Double General Sign of Operation** (or double line) be used to represent the main division line of the problem. As with the **Single Sign of Operation,** the sign should occupy not less than four Braille cells. Examples:

$$\frac{\frac{a+b}{b}}{\frac{c-d}{d}} = \frac{a+b}{b} \times \frac{d}{c-d} = \frac{ad+bd}{bc-bd}$$



This sign may also be used to represent the double line found at the end of balance sheets and financial statements in texts on bookkeeping and commercial arithmetic, as well as the double [25] line at the top of statistical tables.

Ratio Sign : ⠒ (dots 2-5). Examples:

4 : 5 ⠼⠙⠒⠼⠑ x : y ⠭⠒⠽

Proportion : :: : ⠒ ⠒⠒ ⠒ (dots 2-5 and 2-5, 2-5 and 2-5). Examples:

3 : 6 :: 4 : 8 ⠼⠉⠒⠼⠋⠒⠒⠼⠙⠒⠼⠓

x : y :: p : q ⠭⠒⠽⠒⠒⠏⠒⠟

Usages:

In writing lengthy expressions, formulas or equations requiring more than one Braille line, division should be made at a **Sign of Operation,** preferably following the sign. Example:

2,000,000 × 200 = ⠼⠃⠂⠚⠚⠚⠂⠚⠚⠚⠐⠡⠼⠃⠚⠚

400,000,000 ⠼⠙⠚⠚⠂⠚⠚⠚⠂⠚⠚⠚

Although the division of short expressions is not recommended, when it is [26] necessary to split an expression composed of the product of several factors, a **Multiplication Sign** must be inserted before the break. Example:

The expression 15xyz when split after the first letter should be written:

15x ⠼⠁⠑⠐⠭

yz ⠽⠵

If a **Sign of Operation** is used in an expression containing words or abbreviations, the symbol should be preceded and followed by a space. However, when only figures are used (including figures written with **Symbols or Abbreviations of Value or Measurement)** the **Sign of Operation** is written without a space preceding or following. Examples:

3 right angles × 2 = 6 right angles. ⠼⠉⠗⠁⠝⠛⠇⠑⠎⠐⠡⠼⠃⠨⠅⠼⠋⠗⠁⠝⠛⠇⠑⠎

△ ABC = △ XYZ

(See abbreviation for triangle, page 36).

3000 ft. ÷ 20 = 150 ft.

Principal + interest = amount

[27]

$3.00 + $5.00 = $8.00

SYMBOLS OF GROUPING

If possible, expressions within **Symbols of Grouping** should be written complete on one line.

Parentheses ( ) (dots 2-4-6 and 1-3-5). Example:

(x + y)

Brackets [ ] (dots 1-2-3-5-6 and 2-3-4-5-6). Example:

[a — b]

Braces { } (dots 2-3-6 and 3-5-6). Example:

{2x + 5y}

If more than three symbols of grouping are required, the **Literary Parentheses** may be used. The **Vinculum** $\overline{\qquad}$ is not represented in Braille. [28]

Usages:

If an expression composed of the product of several quantities contained within **Symbols of Grouping**

must be split into two Braille lines, the division should come between the **Symbols of Grouping**, and a **Multiplication Sign** should be inserted at the end of the first line. Example:

The expression (a — b) (c — d) when split after the first quantity, should be written:

(a — b)

(c — d)

Examples of Use:

$[x + (3x - 2y) + z] = 12$

$\{2x + [y + (x - y) - z] + 2y\} = 16$

$(x - \{3y - [x - 2z] - x\} + 3z) = 5$

See also rules for **Mixed Numbers**, pages 18-19; **Index Signs**, pages 19-20; **Root Signs**, pages 23-25; **Factorial Sign**, page 28; **Illustration in Combinations**, [29] page 30; and **Integral Sign**, page 32.

FRACTIONS

Fraction Line / ⠌ (dots 3-4). The **Fraction Line** is placed between the numerator and denominator of a fraction. The denominator is written without a **Numeral Sign** when both numerator and denominator are simple numerals. For example:

1/2

$\frac{1}{2x}$

1/b

a/2

a/b

The following way of writing fractions is sometimes used:

1/2 ⠼⠃⠌⠁ or ⠁⠌⠃ 67/89 ⠼⠖⠶⠌⠓⠊

Care should be exercised in using this form, as the last figure of the denominator might be mistaken for a **Sign of Operation** or **Punctuation.** [30]

MIXED NUMBERS

Usages:

The fractional part of a mixed number is joined to the whole number by a **Hyphen.** For example:

$4\frac{7}{8}$ ⠼⠙⠤⠶⠌⠦

Whenever a numerator or a denominator consists of more than one term, it must be inclosed in parenthetical marks. For example:

$\frac{(x + y)}{z}$ ⠐⠣⠭⠬⠽⠐⠜⠌⠵

$\frac{x}{(y + z)}$ ⠭⠌⠐⠣⠽⠬⠵⠐⠜

$\frac{(x - y)}{(p + q)}$ ⠐⠣⠭⠤⠽⠐⠜⠌⠐⠣⠏⠬⠟⠐⠜

$\frac{[a - (c + d)]}{[x + (y - z)]}$ ⠨⠷⠁⠤⠐⠣⠉⠬⠙⠐⠜⠨⠾⠌⠨⠷⠭⠬⠐⠣⠽⠤⠵⠐⠜⠨⠾

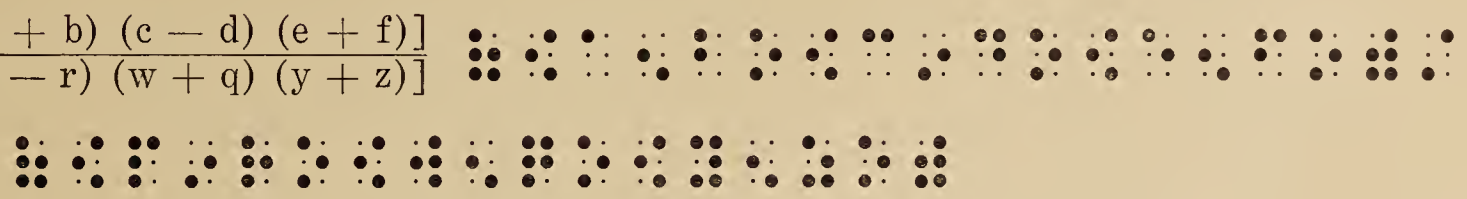
It is usually not necessary to inclose a numerator or a denominator when it is the product of several factors. For example:

$\frac{abcd}{wxyz}$ ⠁⠃⠉⠙⠌⠺⠭⠽⠵

$\frac{(a + b)\ (c - d)}{(p - q)\ (x + y)}$ ⠐⠣⠁⠬⠃⠐⠜⠐⠣⠉⠤⠙⠐⠜⠌⠐⠣⠏⠤⠟⠐⠜⠐⠣⠭⠬⠽⠐⠜

It is, however, sometimes an aid to clearness to inclose a term composed of a large number of factors. For example: [31]

$$\frac{[(a + b)\ (c - d)\ (e + f)]}{[(p - r)\ (w + q)\ (y + z)]}$$

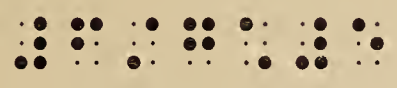


When a fraction is employed as the coefficient of a quantity, it must be inclosed. For example:

$\frac{1}{4}c$ 

$\frac{1}{3}(x + y)$ 

In arithmetic, when a fraction is to be multiplied by a simple numeral, the fraction is not inclosed. For example:

$\frac{6}{7} \times 5$ 

INDEX SIGNS

Numerical Index Sign ⠼ (dot 4). The **Numerical Index Sign** changes all the first ten Braille signs of the alphabet that appear in the index into numerals, unless they are preceded by the **Multiplication Sign** or by the **Letter Sign.** For [32] example:

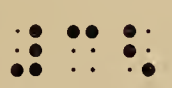
3^5 

x^6 

y^{-1} 

z^{-2} 

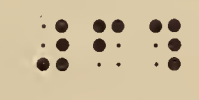
The **Index Sign** is not used for the **Second, Third** and **Fourth Positive Powers,** since special symbols have been assigned to these powers. For example:

3^2 

x^2 

5^3 

y^3 

6^4 

z^4 

$a^4b^4c^4$ 

$x^2y^3z^4$ 

Literal Index Sign ⠸ (dots 4-5). Examples:

7^m 

x^m 

100^n 

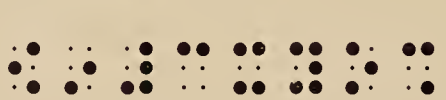
y^n  [33]

Special Usages:

If an exponent relates to a monomial term that is composed of more than one factor, the term is inclosed in **Parentheses.** For example:

$(xy)^2$ 

$(abc)^{-3}$ 

$(-3xy)^3$ 

When a polynomial has an exponent, it is inclosed in **Parentheses.** For example:

$(a + b)^5$ 

$(x + y + z)^{-4}$ 

$(3x + 5y)^2$ 

$(8m - 9n)^3$ 

$(2a^5c^6 - 3b^7d^8 + c^6)^2$ 

A fraction that has an exponent is inclosed in **Parentheses.** For example:

$(x/y)^2$ 

$(a/b)^5$ 

$\left(\begin{matrix} a + b \\ x + y \end{matrix}\right)^4$  [34]

$(r/s)^n$ 

$\left(\frac{x^2}{y^{3-}}\right)^{12}$ 

$\left(\frac{2}{3}\right)^{-5}$ 

A monomial exponent is inclosed in **Braces,** unless it is a simple numeral or a single letter. For example:

x^5 

x^m 

x^{5m} 

y^{-4b} 

z^{abc} 

A polynomial exponent is inclosed in **Braces.** For example:

x^{m+1} 

x^{m+a} 

y^{2m+3} 

y^{bm+c} 

y^{2m+c} 

y^{bm+3} 

$z^{d(a-b)}$ 

$z^{(k+1)1}$ 

A fractional exponent is inclosed in **Braces.** For example:

$x^{½}$  [35]

$x^{a/2}$ 

$x^{-1/b}$ 

$y^{-2/3}$ 

$z^{a/b}$ 

(See also **Special Usages** under **Suffix Signs,** page 21.)

SUFFIX SIGNS

Numerical Suffix Sign ⠐ (dot 6). The **Numerical Suffix Sign** changes all the first ten Braille signs of the alphabet that appear in the suffix into numerals, unless they are preceded by the **Multiplication Sign** or by the **Letter Sign.** For example:

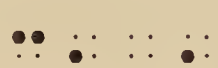
c_6  x_2 

To save space, x_1, x_2, x_3, etc., are sometimes written:

x_1  x_2  x_3 

These symbols are then separated from subsequent **Punctuation Signs** by dot 3, as follows:

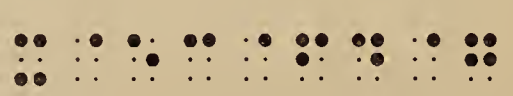
x_1,  [36] x_3: 

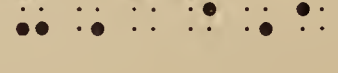
x_2;  x_4. 

Literal Suffix Sign ⠨ (dots 5-6). Examples:

d_m  y_n 

Special Usages: When a factor that has one of the first ten signs of the alphabet as an exponent or a suffix is followed in the same term by a factor that is itself one of the first ten signs of the alphabet, the **Multiplication Sign** is inserted between the two factors. For example:

$x^5c^6d^7$  not 

x_3d_2  not 

PRIME SIGNS

First Prime ′ ⠄ (dots 3-6). Examples:

x′ ⠿⠄⠄ [37]

x′y′z′ ⠿⠄⠿⠄⠽⠄

Second Prime ″ ⠄⠄ (dots 3-6, 3-6). Examples:

x″ ⠿⠄⠄

x″y″ ⠿⠄⠄⠿⠄⠄

ROOT SIGNS

Root Sign ⠩ (dots 1-2-4-5-6). Examples:

$\sqrt[4]{81}$ ⠩⠼⠿⠼⠽⠃

$\sqrt[5]{x}$ ⠩⠼⠽⠭

$\sqrt[6]{-3}$ ⠩⠤⠼⠉

$\sqrt[7]{-y}$ ⠩⠤⠽

$\sqrt[n]{x}$ ⠩⠝⠭

$\sqrt[n]{-y}$ ⠩⠝⠤⠽

Since the **Square Root** and the **Cube Root** are the roots most commonly used, special symbols, which take the place of both **Index** and **Root Sign,** have been assigned to them. For example: [38]

$\sqrt{144}$ ⠜⠼⠙⠙

$\sqrt{-y}$ ⠜⠤⠽

$\sqrt[3]{64}$ ⠌⠼⠋⠙

$\sqrt[3]{y}$ ⠌⠽

$\sqrt[3]{-z}$ ⠌⠤⠵

Special Usages:

The quantity under the **Root Sign** is inclosed in **Parentheses,** unless it is a simple numeral or a single letter, as in the preceding examples. The use of the **Parentheses** is shown in the following illustrations:

$\sqrt{xy}$ 

$\sqrt[5]{5x^2y^3}$ 

$\sqrt[3]{x} \times y$ 

$\sqrt{b^5c^6}$ 

$\sqrt{s^2}$ 

A polynomial, when under a **Root Sign,** is always inclosed in **Parentheses.** For example:

$\sqrt{a + b + c}$  [39]

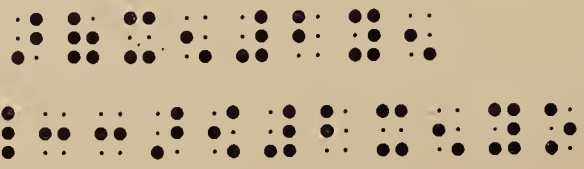
$\sqrt[6]{x^6 + y^7}$ 

Sometimes more than one **Symbol of Grouping** is required. For example:

$\sqrt[3]{5x + \sqrt{4x + 3y} + z} = 3$ 

$\sqrt[4]{4x + \sqrt[3]{5x + \sqrt{4x + 3y} + z} - y} = 2$ 

$\sqrt[5]{9y + \sqrt[3]{5x + \sqrt{x + 6z} + y} + z} = 2$ 

$\sqrt{x + 2y + \sqrt{x^2 - 2xy + y^2}} = \sqrt{2x + y}$ 

When an entire fraction is under a **Root Sign,** it is inclosed in **Parentheses.** For example:

$\sqrt{\frac{2}{3}}$ 

$\sqrt[3]{7\frac{5}{8}}$ 

$\sqrt[4]{\frac{x}{y}}$ 

$\sqrt[6]{\frac{a+b}{x-y}}$ 

Compare the meanings of the following fractions:

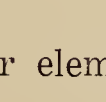
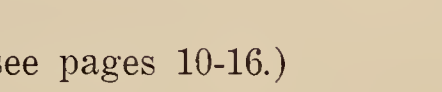
$\frac{\sqrt{3}}{5}$ 

$\sqrt{\frac{3}{5}}$ 

$\frac{3}{\sqrt{5}}$  [40]

ADVANCED SIGNS OF OPERATION

(For elementary mathematical Braille signs, see pages 10-16.)

Difference Between $\eqsim$  (dots 2-5, 3). Examples: $p \eqsim q$ 

Note: The **Letter Signs** and spaces are frequently omitted, as the possibility of a mistake in interpretation is negligible. Example:

$p \eqsim q$ 

Identically Equal To, or **Congruent To** $\equiv$ ⠸⠶ (dots 2-3-5-6, 2-3-5-6). Examples:

$x \times \frac{y}{x} \equiv y$ ⠼⠭⠐⠌⠽⠐⠶⠽

Triangle ABC $\equiv$ Triangle XYZ ⠠⠞⠗⠊⠁⠝⠛⠇⠑ ⠠⠁⠃⠉ ⠶ ⠠⠞⠗⠊⠁⠝⠛⠇⠑ ⠠⠭⠽⠵

The spaces before and after this symbol are frequently omitted when used in algebra. For example: [41]

$x \equiv y$ ⠭⠶⠽

Nearly Equal To $\approx$ ⠸⠒ (dots 2-5, 2-5, 2-5). Example: $p \approx q$ ⠏ ⠸⠒ ⠟

This is frequently written ⠏⠸⠒⠟

Not Equal To $\neq$ ⠨⠅ (dots 2-5, 2-4-5-6). Example: $c \neq d$ ⠉ ⠨⠅ ⠙

This is frequently written ⠉⠨⠅⠙, but the spaces should not be omitted if there is any possibility of the symbols meaning "the ratio of c to wd."

Greater Than $>$ ⠸⠣ (dots 2-5, 1-4). Example: $p > q$ ⠏ ⠸⠣ ⠟

This is frequently written ⠏⠸⠣⠟, except when the symbols might be mistaken for "the ratio of p to cq."

Not Greater Than $\not>$ ⠸⠬ (dots 2-5, [42] 1-4-5-6). Example: $x \not> y$ ⠭ ⠸⠬ ⠽

This is frequently written ⠭⠸⠬⠽

Greater Than or Equal To $\geqslant$ ⠸⠶ (dots 2-5, 1-2-4-5). Example:

$x \geqslant y$ ⠭⠸⠶⠭⠸⠶⠽

This is frequently written ⠭⠶⠽⠸, but care must be taken lest it be interpreted as "the ratio of x to gy."

Less Than $<$ ⠸⠦ (dots 2-5, 3-6). Example: $m < n$ ⠍⠸⠦⠝⠸⠦⠝⠸

This is frequently written ⠭⠶⠽⠸

Not Less Than $\nless$ ⠸⠒ (dots 2-5, 3-4-5-6). Example: $p \nless q$ ⠏⠸⠒⠟⠸⠒⠟⠸

The spaces preceding and following this symbol are sometimes omitted, but care must be exercised. For example: ⠏⠸⠒⠃ may also [43] mean "the ratio of a to 2."

Equal To or Less Than $\leqslant$ ⠸⠣ (dots 2-5, 2-3-5-6). Example: $p \leqslant q$ ⠏⠸⠣⠟⠸⠣⠟⠸

This is sometimes written ⠏⠶⠽⠸, but care must be taken lest it be read as "the ratio of p to factorial q."

Greater Than or Less Than $><$ ⠸⠔ (dots 2-5, 1-3-4-6). Example:

$v><y$ ⠧⠸⠔⠽⠸⠔⠽⠸

This may be written ⠧⠶⠽⠸, but care should be taken lest it be read "the ratio of v to xy."

Greater Than, Equal To, or Less Than $>=<$ ⠒⠶ (dots 2-5, 1-2-3-4-5-6). Example:

$x>=<y$ ⠭⠒⠶⠒⠒⠶⠒⠭⠶

This is often written ⠶⠒⠒⠶ [44]

Varies As $\propto$ ⠒⠳ (dots 2-5, 1-2-3-6). Example: $x \propto y$ ⠭⠒⠳⠒⠒⠳⠒⠽

This is often written ⠶⠒⠒⠳ , but care is necessary to avoid misinterpretation as "the ratio of x to vy."

INFINITY

Infinity ∞ ⠿ (dots 1-2-3-4-5-6). Example: $x \div 0 = \infty$ ⠭⠌⠴⠶⠿

FACTORIALS

Factorial Sign ⠫ ⠒ (dots 2-3-5-6). **The Factorial Sign** changes the first ten signs of the alphabet into numerals, unless they are preceded by the **Multiplication Sign** or by the **Letter Sign.** For example:

⠫4 (that is, $4 \times 3 \times 2 \times 1$) ⠒⠶

⠫3p ⠒⠶⠏

⠫a ⠒⠶⠁ [45]

⠫3b ⠒⠶⠃

⠫n + 2p + 1 ⠒⠶⠝⠬⠏⠬⠼⠁

LOGARITHMS

Logarithm Sign log ⠇⠜ (dots 4-5-6 and 1-2-3). Example:

$\log_{10} 50$ ⠇⠜⠂⠚ or ⠇⠜⠂⠚

Log 50 is sometimes written ⠇⠜⠚, but if the base is indicated, the number must be inclosed between the two parts of the sign. For example:

$\log_e n$ ⠇⠜⠢⠝

log 100 = 2.0000 ⠇⠜⠂⠚⠄⠚⠚⠚⠚

The **Abbreviation "log"** written without a period, is sometimes used to mean "the log of." For example:

log 50 ⠇⠕⠚ log $_e$ n ⠇⠕⠝

The **Abbreviations "colog"** and **"antilog"** are also used in this manner.

Bar Sign $\overline{\ }$ ⠸ (dots 4-5-6). This [46] sign is used to indicate that the characteristic of a logarithm is negative, but the mantissa is positive. For example:

log 0.2 = $\overline{1}$.3010 ⠇⠜⠄⠂⠤⠁⠄⠒⠚⠁⠚

Since this symbol is only used with the integral part of a decimal number there should be no confusion with the sign for a **Recurring Decimal.**

PERMUTATIONS AND COMBINATIONS

Permutations. Examples:

^nP_r (number of **Permutations** of n things taken r together) ⠇⠝⠏⠗

$^5P_4$ ⠇⠢⠏⠲

Combinations. Examples:

$^{n}C_{r}$ (number of **Combinations** of n things taken r together).

$^{10}C_{5}$ [47]

Illustration in **Combinations:**

Out of eight consonants and four vowels, how many combinations of letters can be chosen containing two vowels and three consonants? Solution:

$$^{8}C_{3} \times {}^{4}C_{2} = \frac{\lfloor 8}{\lfloor 3 \quad \lfloor 5} \times \frac{\lfloor 4}{\lfloor 2 \quad \lfloor 2} = 336$$

TRIGONOMETRICAL RATIOS

Sine sin (dots 2-3-5, 2-3-4). Example: sin 30°

Cosine cos (dots 2-3-5, 1-4). Example: cos 2x

Tangent tan (dots 2-3-5, 2-3-4-5). Example: tan Θ

Cotangent cot (dots 2-3-5, 1-2-5-6). Example: cot Θ [48]

Secant sec (dots 2-3-5, 3-6). Example: sec 3x

Cosecant cosec (dots 2-3-5, 1-2-6). Example: cosec 45°

The following **Abbreviations** may also be substituted for the symbols given above:

Sine sin  (dots 2-3-4, 3-5).

Cosine cos  (dots 1-4, 1-3-5, 2-3-4).

Tangent tan  (dots 2-3-4-5, 1, 1-3-4-5).

Cotangent cot  (dots 1-4, 1-3-5, 2-3-4-5) or  (dots 1-4, 2-3-4-5, 1-3-4-5).

Secant sec  (dots 2-3-4, 1-5, 1-4).

Cosecant cosec  (dots 1-4, [49] 1-3-5, 2-3-4, 1-5, 1-4) or

csc  (dots 1-4, 2-3-4, 1-4).

The **Index** of the power to which a **Trigonometrical Ratio** is raised is inserted between the sign of the ratio and the angle. For example:

sin ^{3} x 

cos ^{4} x 

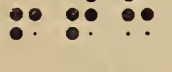
tan ^{5} x 

cosec ^{2} x 

HYPERBOLIC FUNCTIONS

Hyperbolic Sine sinh  (dots 2-3-5, 2-3-4, 1-2-5).

Hyperbolic Cosine cosh  (dots 2-3-5, 1-4, 1-2-5).

Hyperbolic Tangent tanh  (dots 2-3-5, 2-3-4-5, 1-2-5).

Hyperbolic Contangent coth  (dots 2-3-5, 1-2-5-6, 1-2-5). [50]

Hyperbolic Secant sech  (dots 2-3-5, 3-6, 1-2-5).

Hyperbolic Cosecant cosech  (dots 2-3-5, 1-2-6, 1-2-5).

The following **Abbreviations** may also be substituted for the symbols given above:

Hyperbolic Sine sinh ⠓⠊⠝ (dots 2-3-4, 3-5, 1-2-5).

Hyperbolic Cosine cosh ⠉⠕⠎⠓ (dots 1-4, 1-3-5, 2-3-4, 1-2-5).

Hyperbolic Tangent tanh ⠞⠁⠝⠓ (dots 2-3-4-5, 1, 1-3-4-5, 1-2-5).

Hyperbolic Cotangent coth ⠉⠕⠞⠓ (dots 1-4, 1-3-5, 2-3-4-5, 1-2-5) or ctnh ⠉⠞⠝⠓ (dots 1-4, 2-3-4-5, 1-3-4-5, 1-2-5).

Hyperbolic Secant sech ⠎⠑⠉⠓ (dots 2-3-4, 1-5, 1-4, 1-2-5).

Hyperbolic Cosecant cosech ⠉⠕⠎⠑⠉⠓ (dots 1-4, 1-3-5, 2-3-4, [51] 1-5, 1-4, 1-2-5) or csch ⠉⠎⠉⠓ (dots 1-4, 2-3-4, 1-4, 1-2-5).

Note: Care should be taken not to use the **ch** or **sh** or **th** signs in the above **Abbreviations.**

SYMBOLS FOR CALCULUS

Differential Coefficient: $\frac{dx}{dt}$ ⠙⠭⠱⠙⠞

Second Differential Coefficient: $\frac{d^2x}{dt^2}$ ⠙⠘⠼⠭⠱⠙⠞⠘⠼

Integral Sign ∫ ⠮ (dots 3, 2-3-4). Examples:

∫f(x)dx ⠮⠋⠐⠣⠭⠜⠙⠭

$\int_a^b f(x)dx$ ⠮⠰⠁⠻⠃⠋⠐⠣⠭⠜⠙⠭

[52] ABBREVIATIONS

Abbreviations employed in ink-print mathematical texts may be used in Braille. In addition, the list below may be used in texts **more advanced than arithmetic.** Some texts may employ abbreviations which vary slightly from the form given here, and the **Abbreviation Point** may or may not be used. Abbreviations should be preceded and followed by a space:

adjacent

alternate ~

altitude

angle ∠

angular (dots 4-5-6, 1)

another

arc* ⌒

axiom

base (dots 4-5, 1-2)

bisect (dots 4-5-6, 1-2)

call (dots 4-5, 1-4)

center (dots 4-5-6, 2-5)

centric (dots 4-5-6, 2-5, 1-4)

change (dots 4, 1-6)

chord (dots 4-5-6, 1-6)

circle ○

circular measure

circum

circumference ⊙

complement

complementary

condition

consider

constant

[53]

construct

construction

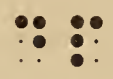
corollary

correspond

curve

definition

*Not an abbreviation.

depend 

describe 

diagonal  [54]

diameter  (dots 4-5, 1-4-5)

differ 

different 

direction  (dots 4-5-6, 1-4-5)

divide 

equal 

equation 

equidistant 

equilateral 

example  (dots 4-5-6, 1-5)

exercise 

expression 

exterior 

external 

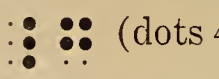
figure 

follow  (dots 4-5-6, 1-2-4)

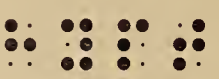
general 

geometrical 

geometry 

graph  (dots 4-5-6, 1-2-4-5)

horizontal 

hypotenuse 

hypothesis 

identity  [55]

infinite 

inscribe 

intercept 

interior 

internal 

intersect 

intersection 

isosceles 

length 

line  (dots 4-5-6, 1-2-3)

logarithm  (See also page 29).

magnitude

maximum

measure

meter

middle (dots 4-5, 1-3-4)

negative

number (dots 4-5-6, 1-3-4-5)

opposite

origin (dots 4-5-6, 1-3-5)

parallel ∥

parallelogram

particular (dots 5, 1-2-3-4, 1-2-3-5) [56]

perpendicular ⊥

plane

point (dots 4-5-6, 1-2-3-4)

portion

position

positive

postulate

problem

projection

property

proportion (See also page 15).

proposition

quadrilateral

quantity

quantities

radians

radius

ratio (dots 4-5-6, 1-2-3-5) (See also page 15).

rectangle ▭

rectilineal

rectilinear

represent

require

required [57]

respective

respectively

right (dots 5, 1-2-3-5)

right angle ∟ (dots 5, 1-2-3-5, 4-5-6, 1)

satisfy

side (dots 4-5, 2-3-4)

similar

square □

straight*

supplement

supplementary

surface

symmetrical

symmetry

tangent (geometrical) (See also form page 30, and form , page 31).

theorem

therefore

together

triangle △

trigonometrical [58]

value (dots 4-5-6, 1-2-3-6)

vertex

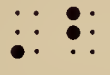
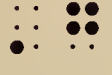
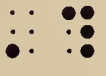
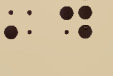
vertical

volume

wherefore

*Also, when grade 2 is not used.

GREEK LETTERS

Alpha	A	α		Nu	N	ν	
Beta	B	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	O	o	
Delta	Δ	δ		Pi	Π	π	
Epsilon	E	ϵ		Rho	P	ρ	
Zeta	Z	ζ		Sigma	Σ	σ or ς	
Eta	H	η		Tau	T	τ	
Theta	Θ	θ		Upsilon	Υ	υ	
Iota	I	ι		Phi	Φ	ϕ	
Kappa	K	κ		Chi	X	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	M	μ		Omega	Ω	ω	

Date Due
